Property Modifiers

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Abstract. In this paper, we deal with property modifiers defined as functions that associate a given root property P with a modified property $[MP]$. Property modifiers typically divide into four kinds, namely intersective, subsective, privative and modal. Here we do not deal with modal modifiers like alleged, which appear to be well-nigh logically lawless, because, for instance, an alleged assassin is or is not an assassin. The goal of this paper is to logically define the three remaining kinds of modifiers. Furthermore, we introduce the rule of pseudo-detachment as the rule of left subsectivity to replace the modifier M in the premise by the property M^* in the conclusion, and prove that this rule is valid for all kinds of modifiers. Furthermore, it is defined in a way that avoids paradoxes like that a small elephant is smaller than a large mouse.

Key words: Property modifier, subsective, intersective, privative, the rule of pseudo-detachment, Transparent intensional logic, TIL, intensional essentialism

1 Introduction

We introduce a logic of property modifiers modelled as a mapping from properties to properties, such that the result of the application of a modifier to a property is another property. This is because the result of modification does not depend on the state of the world, nor on time. For instance, if one applies the modifier *Skilful* to the property *Surgeon*, they obtain the property of being a skilful surgeon. The conception of modifiers presented here goes along the lines introduced in Duží et.al. [2, §4.4]. The novel contribution of this paper is a new definition of subsective and privative modifiers in terms of *intensional essentialism*.

As a starting point, here is a standard taxonomy of the three kinds of modifiers, with rigorous definitions coming afterwards. Let the extension of a property P be $|P|$[1], M standing for a modifier, M^* for the property corresponding to a

[1] Extensionalization of properties will be explained below; it corresponds to the application of a property to empirical indexes such as world and time.

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modifier[2].

Intersective. "If a is a *round* peg, then a is round and a is a peg."

$M_i P(a) \therefore M^*(a) \wedge P(a)$.
Necessarily, $|M_i P| = |M^*| \cap |P|$.

Necessarily, i.e. in all worlds and times, the set of round pegs equals to the intersection of the sets of round objects and pegs.

Note that we cannot transfer M_i from the premise to the conclusion. The reason is that a modifier cannot also occur as a predicate; these are objects of different types. Hence M^* instead of just M_i.

Subsective. "If a is a *skilful* surgeon, then a is a surgeon."

$M_s P(a) \therefore P(a)$.
Necessarily, $|M_s P| \subseteq |P|$.

Necessarily, i.e. in all worlds and times, the set of skilful surgeons is a subset of the set of surgeons.

The major difference between subsective and intersective modification is that this sort of argument: $M_s P(a), Q(a) \therefore M_s Q(a)$ is *not* valid for subsective modifiers. Tilman may be a skilful surgeon, and he may be a painter too, but this does not make him a skilful painter. Scalar adjectives like 'small', 'big' or 'skilful' represent subsective modifiers. On the other hand, to each intersective modifier M_i there is a unique 'absolute' property M^* such that if a is an $M_i P$ then a is M^* not only as a P but absolutely.

Privative. "If a is a *forged* banknote, then a is not banknote."

$M_p P(a) \therefore \neg P(a)$.
Necessarily, $|M_p P| \cap |P| = \varnothing$.

Necessarily, i.e. in all worlds and times, the intersection of the set of forged banknotes and banknotes is empty.

Modifiers are intersective, subsective and privative *with respect to a property* P. One and the same modifier can be intersective with respect to a property P and privative with respect to another property Q. For instance, a wooden table is wooden and is a table, but a wooden horse is not a horse. We leave aside the question whether there are modifiers privative with respect to any property. Most probably, yes, modifiers like *faked*, *forged*, *false* appear to be privative with

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[2] The corresponding property M^* is defined below by the rule of pseudo-detachment. It is the property M (*something*), where in case of intersective modifiers M^* is an 'absolute' property. Hence a round peg is round not only as a peg, but absolutely. See Jespersen [6] for details.

respect to any property. Yet this issue is irrelevant to the main goal of this paper, which is to define the *rule of pseudo-detachment* (*PD*) and prove its validity for *any* kind of modifiers.

The rest of the paper is organised as follows. Section 2 introduces the fundamentals of our background theory TIL necessary to deal with property modifiers, which is the issue we deal with in Section 3. Here in Section 3.1 the difference between non-subsective and subsective modifiers is defined, followed by the rule of pseudo-detachment defined in Section 3.2. Concluding remarks can be found in Section 4.

2 Basic Notions of TIL

Tichý's TIL comes with *procedural semantics*, which means that we explicate meanings of language expressions as abstract procedures encoded by the expressions. Tichý defined six kinds of procedures as the so-called *constructions*[3]. Here we need only four of them, leaving aside Single and Double Execution.

Definition 1 (*construction*).

(i) *Variables* x, y, ... are *constructions* that construct objects (elements of their respective ranges) dependently on a valuation v; they *v-construct*.
(ii) Where X is an object whatsoever (even a *construction*), ^0X is the *construction Trivialization* that constructs X without any change in X.
(iii) Let X, Y_1,...,Y_n be arbitrary *constructions*. Then *Composition* $[X\ Y_1...Y_n]$ is the following *construction*. For any v, the *Composition* $[X\ Y_1...Y_n]$ is v-improper if at least one of the *constructions* X, Y_1,...,Y_n is v-improper, or if X does not *v-construct* a function that is defined at the n-tuple of objects *v-constructed* by Y_1,...,Y_n . If X does *v-construct* such a function then $[X\ Y_1...Y_n]$ *v-constructs* the value of this function at the n-tuple.
(iv) (λ-)*closure* $[\lambda x_1...x_m Y]$ is the following *construction*. Let x_1, x_2, ..., x_m be pairwise distinct variables and Y a *construction*. Then $[\lambda x_1...x_m Y]$ *v-constructs* the function f that takes any members B_1,...,B_m of the respective ranges of the variables x_1, ..., x_m into the object (if any) that is $v(\mathrm{B}_1/x_1$,...,
$\mathrm{B}_m/x_m)$-*constructed* by Y, where $v(\mathrm{B}_1/x_1,...,\mathrm{B}_m/x_m)$ is like v except for assigning B_1 to x_1,...,B_m to x_m.
(v) Nothing is a *construction*, unless it so follows from (i) through (iv).

In Tichý's TIL constructions are objects *sui generis*, so that we can have constructions of constructions, constructions of functions, functions, and functional values in TIL stratified ontology. To keep track of the traffic between multiple logical strata, the ramified type hierarchy is needed. The type of first-order objects includes all non-procedural objects. Therefore, it includes not only the standard objects of individuals, truth-values, sets, etc., but also functions defined on possible worlds (i.e., the intensions germane to possible-world semantics). The type of second-order objects includes constructions of

[3] See Tichý [7, Chapters 4, 5] or Duží, Jespersen & Materna [2, §1.3]

first-order objects and functions with such constructions in their domain or range. The type of third-order objects includes constructions of first- and second-order objects and functions with such constructions in their domain or range. And so on, ad infinitum. Yet, for the purposes of this paper we need just the simple theory of types. Hence, we define.

Definition 2 (***simple theory of types***). Let *B* be a *base*, where a base is a collection of pair-wise disjoint, non-empty sets. Then:

i) Every member of *B* is an elementary *type of order 1 over B.*
ii) Let α, β_1, ..., β_m ($m > 0$) be types of order 1 over *B*. Then the collection ($\alpha\,\beta_1$ ... β_m) of all *m*-ary partial mappings from $\beta_1 \times ... \times \beta_m$ into α is a functional *type of order 1 over B.*
iii) Nothing is a *type of order 1 over B* unless it so follows from (i) and (ii).

For the purposes of natural-language analysis, we are assuming the following base of ground types:

o : the set of truth-values {**T, F**};
ι : the set of individuals (the universe of discourse);
τ : the set of real numbers (doubling as discrete times);
ω : the set of logically possible worlds (the logical space).

We model sets and relations by their characteristic functions. Thus, for instance, (oι) is the type of a set of individuals, while (o$\iota\iota$) is the type of a relation-in-extension between individuals. Empirical expressions denote *empirical conditions* that may or may not be satisfied at the particular world/time pair of evaluation. We model these empirical conditions as possible-world-semantic (*PWS*) *intensions*. PWS intensions are entities of type ($\beta\omega$): mappings from possible worlds to an arbitrary type β. The type β is frequently the type of the *chronology* of α-objects, i.e., a mapping of type ($\alpha\tau$). Thus α-intensions are frequently functions of type (($\alpha\tau$)ω), abbreviated as '$\alpha_{\tau\omega}$'. Extensional entities are entities of a type α where $\alpha \neq (\beta\omega)$ for any type β. Where w ranges over ω and t over τ, the following logical form essentially characterizes the logical syntax of empirical language: $\lambda w \lambda t\, [\ldots w \ldots t \ldots]$.

Examples of frequently used PWS intensions are: propositions of type $o_{\tau\omega}$, properties of individuals of type $(o\iota)_{\tau\omega}$, binary relations-in-intension between individuals of type $(o\iota\iota)_{\tau\omega}$, individual offices (or roles) of type $\iota_{\tau\omega}$.

Modifiers of individual properties are extensional entities of type $((o\iota)_{\tau\omega}(o\iota)_{\tau\omega})$.

Logical objects like *truth-functions* and *quantifiers* are extensional: $\wedge$ (conjunction), $\vee$(disjunction) and $\supset$ (implication) are of type (ooo), and $\neg$ (negation) of type (oo). Quantifiers $\forall^\alpha$, $\exists^\alpha$ are type-theoretically polymorphic total functions of type (o(oα)), for an arbitrary type α, defined as follows. The *universal quantifier* $\forall^\alpha$ is a function that associates a class *A* of α-elements with **T** if *A* contains all elements of the type α, otherwise with **F**. The *existential quantifier* $\exists^\alpha$ is a function that associates a class A of α-elements with **T** if A is a non-empty class, otherwise with **F**.

Below all type indications will be provided outside the formulae in order not to clutter the notation. Moreover, the outermost brackets of the Closure will be omitted whenever no confusion can arise. Furthermore, 'X/α' means that an object X is (a member) of type α. '$X \to \alpha$' means that the construction X is typed to v-construct an object of type α, if any. Throughout, it holds that the variables $w \to \omega$ and $t \to \tau$. If $C \to \alpha_{\tau\omega}$ then the frequently used Composition $[[C\ w]\ t]$, which is the intensional descent (a.k.a. extensionalization) of the α-intension v-constructed by C, will be encoded as 'C_{wt}'. Whenever no confusion arises, we use traditional infix notation without Trivialisation for truth-functions and the identity relation, to make the terms denoting constructions easier to read. Thus, for instance, instead of '$[^0\wedge\ [^0{=}\ [^0{+}\ ^02\ ^05]\ ^07]\ [^0{\supset}\ p\ q]]$' we usually simply write '$[[[^0{+}\ ^02\ ^05] = {}^07] \wedge [p \supset q]]$'.

3 Property Modifiers and Intensional Essentialiasm

3.1 Privative vs Subsective Modifiers

The fundamental distinction among modifiers is typically considered to be one between the *subsectives* and the *non-subsectives*. The former group consists of the *pure subsectives* and the *intersectives*. The latter group consists of the modals and the privatives. Since we are not dealing with modal modifiers here, we now want to define the distinction between *subsectives* and *privatives*. At the outset of this paper this distinction between modifiers subsective (M_s) and privative (M_p) with respect to a property P has been characterized by the rules of the right subsectivity as follows:

$$M_sP(a) \therefore P(a)$$

$$M_pP(a) \therefore \neg P(a)$$

Now we have the technical machinery at our disposal to define these modifiers in a rigorous way. To this end, we apply the logic of intensions based on the notions of *requisite* and *essence* of a property, which amounts to *intensional essentialism*[4]. The idea is this. Every property has a host of other properties necessarily associated with it. For instance, the property of being a bachelor is associated with the properties of being a man, being unmarried, and many others. Necessarily, if a happens to be a bachelor then a is a man and a is unmarried. We call these adjacent properties *requisites* of a given property.

The requisite relations *Req* are a family of relations-in-extension between two intensions, so they are of the polymorphous type $(o\,\alpha_{\tau\omega}\,\beta_{\tau\omega})$, where possibly α

[4] In contrast to *individual anti-essentialism*: no individual has a non-trivial empirical property necessarily. In other words, only trivial properties like being self-identical, being identical to a or b, etc., are necessarily ascribed to an individual a. For details see Duží et al. [2, §4.2], and also Cmorej [1].

= β. Infinitely many combinations of *Req* are possible, but for our purpose we will need the following one: $Req/(o(o\iota)_{\tau\omega}(o\iota)_{\tau\omega})$; a property of individuals is a requisite of another such property. TIL embraces *partial functions*[5]. Partiality gives rise to the following complication. The requisite relation obtains analytically necessarily, i.e., for all worlds w and times t, and so the values at the $\langle w,t\rangle$-pairs of particular intensions are irrelevant. But the values of properties are isomorphic to characteristic functions, and these functions are amenable to truth-value gaps. For instance, the property of having stopped smoking comes with a bulk of requisites like, e.g., the property of being a former smoker. If a never smoked, then the proposition that a stopped smoking comes with a truth-value gap, because it can be neither true nor false that a stopped or did not stop smoking. Thus, the predication of such a property P of a may also fail, causing $[^0P_{wt}\ ^0a]$ to be v-improper. There is a straightforward remedy, however, namely the propositional property of being true at $\langle w,t\rangle$: $True/(oo_{\tau\omega})_{\tau\omega}$. Given a proposition v-constructed by X, $[^0True_{wt}\ X]$ v-constructs **T** if the proposition presented by X is true at $\langle w,t\rangle$; otherwise (i.e., if the proposition constructed by X is false or else undefined at $\langle w,t\rangle$) **F**. Thus we define:

Definition 3 (*requisite relation between ι-properties*). Let P, Q be constructions of individual properties; $P, Q \rightarrow (o\,\iota)_{\tau\omega}$; $x \rightarrow \iota$. Then

$$[^0Req\ Q\ P] \ = \ \forall w\forall t\,[\,\forall x\,[[^0True_{wt}\ \lambda w\lambda t\,[P_{wt}\ x]] \supset [^0True_{wt}\ \lambda w\lambda t\,[Q_{wt}\ x]]]].$$

Next, we are going to define the essence of a property. Our essentialism is based on the idea that since no purely contingent property can be essential of any individual, essences are borne by intensions rather than by individuals exemplifying intensions[6]. Hence, our essentialism is based on the requisite relation, couching essentialism in terms of a priori interplay between properties, regardless of who or what exemplifies a given property. *Intensional essentialism* is technically an algebra of individually necessary and jointly sufficient conditions for having a certain property (or other sort of intension). The $\langle w,t\rangle$-relative extensions of a given property are irrelevant, as we said.

Definition 4 (*essence of a property*). Let $p, q \ \rightarrow \ (o\iota)_{\tau\omega}$ be constructions of individual properties, and let $Ess/((o(o\iota)_{\tau\omega})(o\iota)_{\tau\omega})$, i.e. a function assigning to a given property p the set of its requisites defined as follows:

$$^0Ess = \lambda p\lambda q\,[^0Req\ q\ p]$$

Then the essence of a property p is the set of its requisites: $[^0Ess\ p] = \lambda q\,[^0Req\ q\ p]$

[5] See Duží et al. [2, 276-78] for philosophical justification of partiality despite the associated technical complications.

[6] By 'purely contingent intension' we mean an intension that is not a constant function and does not have an essential core (e.g. the property of having exactly as many inhabitants as Prague is necessarily exemplified by Prague).

Each property has (possibly infinitely) many requisites. The question is, how do we know which are the requisites of a given property? The answer requires an *analytic definition* of the given property, which amounts to the specification of its essence. For instance, consider the property of being a bachelor. If we define this property as the property of being an unmarried man, then the properties of being unmarried and being a man are among the requisites of the property of being a bachelor. Thus, the sentence "bachelors are unmarried men" comes out analytically true:

$$\forall w \forall t [\forall x [[{}^0Bachelor_{wt}\, x] \supset [[{}^0Unmarried\ {}^0Man]_{wt}\, x]]].$$

And since the modifier *Unmarried* is intersective, it also follows that necessarily, each bachelor is unmarried and is a man:

$$\forall w \forall t [\forall x [[{}^0Bachelor_{wt}\, x] \supset [[{}^0Unmarried'_{wt}\, x] \wedge [{}^0Man_{wt}\, x]]]].$$

Note, however, that $Unmarried'/(o\iota)_{\tau\omega}$ and $Unmarried/((o\iota)_{\tau\omega}(o\iota)_{\tau\omega})$ are entities of different types. The former is a property of individuals uniquely assigned to the latter, which is an intersective modifier.

With these definitions in place, we can go on to compare two kinds of *subsectives* against privatives[7]. Since these modifiers change the essence of the root property, we need to compare the essences, that is sets of properties, of the root and modified property. To this end, we apply the set-theoretical relations of be-ing a *subset* and a *proper subset* between sets of properties, and the *intersection* operation on sets of properties, defined as follows.

Let $\pi = (o\iota)_{\tau\omega}$, for short, $\subseteq, \subset\ /(o(o\pi)(o\pi))$, and let $a, b \rightarrow_v (o\pi); x \rightarrow_v \pi$. Then

$$\begin{gathered} {}^0\!\subseteq\, = \lambda ab\, [{}^0\forall \lambda x\, [a\, x] \supset [b\, x]] \\ {}^0\!\subset\, = \lambda ab\, [[{}^0\forall \lambda x\, [[a\, x] \supset [b\, x]]] \wedge \neg [a = b]] \end{gathered}$$

Furthermore, the *intersection* function $\cap/((o\pi)(o\pi)(o\pi))$ is defined on sets of properties in the usual way: ${}^0\cap = \lambda ab\, \lambda x\, [[a\, x] \wedge [b\, x]]$. In what follows we will use classical (infix) set-theoretical notation for any sets A, B; hence instead of '$[{}^0\!\subseteq\ A\, B]$' we will write '$[A \subseteq B]$', and instead of '$[{}^0\cap\, A\, B]$' we will write '$[A \cap B]$'.

Definition 5 (*subsective vs. privative modifiers*).

- A modifier M is *subsective* with respect to a property P iff

$$[{}^0Ess\ P] \subseteq [{}^0Ess\ [M\, P]]$$

- A modifier M is *non-trivially subsective* with respect to a property P iff

$$[{}^0Ess\ P] \subset [{}^0Ess\ [M\, P]]$$

[7] Since *intersective* modification is a special kind of *subsective* modification, we are disregarding *intersectives* not to clutter the exposition. Intersectives are controlled by the same rule of *right subsectivity* that applies to the subsectives.

■ A modifier M is *privative* with respect to a property P iff

$$[[{}^0Ess\,P] \cap [{}^0Ess\,[M\,P]]] \neq \varnothing \wedge$$
$${}^0\exists\lambda p\,[[[{}^0Ess\,P]\,p] \wedge [[{}^0Ess\,[M\,P]]\,\lambda w\lambda t\,[\lambda x\,\neg[p_{wt}x]]]]$$

Remark. We distinguish between *subsective* and *non-trivially subsective* modifiers, because among subsectives there are also trivial subsectives. A modifier M is *trivially subsective* with respect to P iff the modified property $[MP]$ has exactly the same essence as the property P. These modifiers are trivial in that the modification has no effect on the modified property and so might just as well not have taken place. For instance, there is no semantic or logical (but perhaps rhetorical) difference between the property of being a leather and the property of being a *genuine* leather. Trivial modifiers such as *genuine, real, actual* are pure subsectives: genuine leather things are not located in the intersection of leather things and objects that are genuine, for there is no such property as being genuine, pure and simple[8].

Example. The modifier $Wooden/((o\iota)_{\tau\omega}(o\iota)_{\tau\omega})$ is subsective with respect to the property of being a table, $Table/(o\iota)_{\tau\omega}$, but privative with respect to the property of being a horse, $Horse/(o\iota)_{\tau\omega}$. Of course, a wooden table is a table, but the essence of the property $[{}^0Wooden\,{}^0Table]$ is enriched by the property of being wooden. This property is a requisite of the property of being a wooden table, but it is not a requisite of the property of being a table, because tables can be instead made of stone, iron, etc.

$$[{}^0Ess\,{}^0Table] \subset [{}^0Ess\,[{}^0Wooden\,{}^0Table]].$$

But a wooden horse is not a horse. The modifier *Wooden*, the same modifier that just modified *Table*, deprives the essence of the property of being a horse, $Horse/(o\iota)_{\tau\omega}$, of many requisites, for instance, of the property of being an animal, having a bloodstream, a heartbeat, etc. Thus, among the requisites of the property $[{}^0Wooden\,{}^0Horse]$ there are properties like *not being a living thing, not having a bloodstream, etc.*, which are contradictory (not just contrary) to some of the requisites of the property *Horse*. On the other hand, the property $[{}^0Wooden\,{}^0Horse]$ shares many requisites with the property of being a horse, like the outline of the body, having four legs, etc., and has an additional requisite of being made of wood. We have:

$$[[{}^0Ess\,{}^0Horse] \cap [{}^0Ess\,[{}^0Wooden\,{}^0Horse]]] \neq \varnothing \wedge$$
$$[[[{}^0Ess\,{}^0Horse]\,{}^0Living\_thing] \wedge$$
$$[[{}^0Ess\,[{}^0Wooden\,{}^0Horse]]\,\lambda w\lambda t\,[\lambda x\,\neg[{}^0Living\_thing\,x]]]] \wedge$$
$$[[[{}^0Ess\,{}^0Horse]\,{}^0Blood] \wedge$$
$$[[{}^0Ess\,[{}^0Wooden\,{}^0Horse]]\,\lambda w\lambda t\,[\lambda x\,\neg[{}^0Blood\,x]]]] \wedge$$
etc.

A modifier M is *privative* with respect to a property P iff the modified property $[MP]$ lacks at least one, *but not all*, of the requisites of the property P.

[8] Iwańska [5, 350] refers to 'ideal', 'real', 'true', and 'perfect' as *type-reinforcing* adjectives, which seems to get the pragmatics right of what are semantically pleonastic adjectives.

However, in this case we cannot say that the essence of the property $[MP]$ is a proper subset of the essence of the property P, because the modified property $[MP]$ has at least one other requisite that does not belong to the essence of P, because it contradicts to some of the requisites of P. Hence, M is privative with respect to property P iff the essence of property $[MP]$ has a non-empty intersection with the essence of the property P, and this intersection is a *proper* subset of both the essences of P and of $[MP]$. For instance, a forged banknote has *almost* the same requisites as does a banknote, but it has also another requisite, namely the property of being forged with respect to the property of being a banknote.

As a result, if M_p is privative with respect to the property P, then the modified property $[M_pP]$ and the property P are contrary rather than contradictory properties:

$$\forall w \forall t\, \forall x[[[M_pP]_{wt}\, x] \supset \neg[P_{wt}\, x]] \wedge \exists w \exists t\, \exists x[\neg[[M_pP]_{wt}\, x] \wedge \neg[P_{wt}\, x]]$$

It is not possible for x to co-instantiate $[M_pP]$ and P, and possibly x instantiates neither $[M_pP]$, nor P.

3.2 The Rule of Pseudo-detachment

The issue we are going to deal with now is left subsectivity[9]. We have seen that the principle of left subsectivity is trivially (by definition) valid for intersective modifiers. If Jumbo is a yellow elephant, then Jumbo is yellow. Yet how about the other modifiers? If Jumbo is a small elephant, is Jumbo small? If you factor out *small* from *small elephant*, the conclusion says that Jumbo is small, period. Yet this would seem a strange thing to say, for something appears to be missing: Jumbo is a small *what*? Nothing or nobody can be said to be small or forged, skilful, temporary, larger than, the best, good, notorious, or whatnot, without any sort of qualification. A complement providing some sort of qualification to provide an answer to the question, 'a … *what*?' is required. We are going to introduce now the rule of pseudo-detachment that is valid for all kinds of modifiers including subsective and privative ones. The idea is simple. From *a* is an *MP* we infer that *a* is an *M-with respect to something*.

For instance, if the customs officers seize a forged banknote and a forged passport, they may want to lump together all the forged things they have seized that day, abstracting from the particular nature of the forged objects. This lumping together is feasible only if it is logically possible to, as it were, abstract *forged* from *a being a forged A* and *b being a forged B* to form the new predications that *a is forged* (something) and that *b is forged* (something), which are subsequently telescoped into a conjunction.

Gamut (the Dutch equivalent of Bourbaki) claims that if Jumbo is a small elephant, then it does not follow that Jumbo is small [3, §6.3.11]. We are going to show that the conclusion does follow. The rule of pseudo-detachment (*PD*)

[9] In this section, we partly draw on material from Duží et.al. [2, §4.4].

validates a certain inference schema, which on first approximation is formalized as follows:

$$(PD) \qquad \frac{a \text{ is an } MP}{a \text{ is an } M^*}$$

where 'a' names an appropriate subject of predication while 'M' is an adjective and 'P' a noun phrase compatible with a.

The reason why we need the rule of pseudo-detachment is that M as it occurs in MP is a *modifier* and, therefore, cannot be transferred to the conclusion to figure as a *property*. So no actual detachment of M from MP is possible, and Gamut is insofar right. But (PD) makes it possible to replace the modifier M by the property M^* compatible with a to obtain the conclusion that a is an M^*. (PD) introduces a new property M^* 'from the outside' rather than by obtaining M 'from the inside', by extracting a part from a compound already introduced. The temporary rule above is incomplete as it stands; here is the full pseudo-detachment rule, SI being substitution of identicals (Leibniz's Law)[10], EG existential generalization.

(1)	a is an MP	assumption
(2)	a is an (M something)	1, EG
(3)	M^* is the property (M something)	definition
(4)	a is an M^*	2,3, SI

To put the rule on more solid grounds of TIL, let $\pi = (o\iota)_{\tau\omega}$ for short, $M \to (\pi\pi)$ be a modifier, $P \to \pi$ an individual property, $[MP] \to \pi$ the property resulting from applying M to P, and let $[MP]_{wt} \to_v (o\iota)$ be the result of extensionalizing the property $[MP]$ with respect to a world w and time t to obtain a set, in the form of a characteristic function, applicable to an individual $a \to \iota$. Further, let $= /(o\pi\pi)$ be the identity relation between properties, and let $p \to_v \pi$ range over properties, $x \to_v \iota$ over individuals. Then the *proof* of the rule is this:

1.	$[[MP]_{wt}\, a]$	assumption
2.	$\exists p\, [[Mp]_{wt}\, a]$	1, $\exists$I
3.	$[\lambda x\, \exists p\, [[Mp]_{wt}\, x]\, a]$	2, λ-expansion
4.	$[\lambda w' \lambda t'\, [\lambda x\, \exists p\, [[Mp]_{w't'}\, x]]_{wt}\, a]$	3, λ-expansion
5.	$M^* = \lambda w' \lambda t'\, [\lambda x\, \exists p\, [[Mp]_{w't'}\, x]]$	definition
5.	$[M^*_{wt}\, a]$	4, 5, SI

Any valuation of the free occurrences of the variables w, t that makes the first premise true will also make the second, third and fourth steps true. The fifth premise is introduced as valid by definition. Hence, any valuation of w, t that makes the first premise true will, together with the step five, make the conclusion true.

(PD), dressed up in full TIL notation, is this[11]:

[10] More precisely, substitution of identical properties.

[11] As mentioned above, in case of the modifier M being intersective, the property M^* is unique for any p. For details see Jespersen [6].

$$(PD)\qquad \frac{\begin{array}{c}[[MP]_{wt}\, a]\\ [M^{*} = \lambda w'\lambda t'\, [\lambda x\, \exists p\, [[Mp]_{w't'}\, x]]]\end{array}}{[M^{*}_{wt}\, a]}$$

Additional type: $\exists/(o(o\pi))$.
Here is an instance of the rule.

(1) *a* is forged banknote
(2) forged* is the property of being a forged something
(3) *a* is forged*.

The schema extends to all (appropriately typed) objects. For instance, let the inference be, "Geocaching is an exciting hobby; therefore, geocaching is exciting". Then a is of type π, $P \rightarrow (o\pi)_{\tau\omega}$, $M \rightarrow ((o\pi)_{\tau\omega}(o\pi)_{\tau\omega})$, and $M^{*} \rightarrow (o\pi)_{\tau\omega}$.
Now it is easy to show why this argument must be valid:

$$\frac{\text{John has a forged banknote and a forged passport}}{\text{John has two forged things.}}$$

$$\frac{\frac{\begin{array}{c}\lambda w\lambda t\, \exists xy\, [{}^{0}Have_{wt}\, {}^{0}John\, x] \wedge [{}^{0}Have_{wt}\, {}^{0}John\, y] \wedge\\ [[{}^{0}Forged\, {}^{0}Banknote]_{wt}\, x] \wedge [[{}^{0}Forged\, {}^{0}Passport]_{wt}\, y] \wedge [{}^{0}\neq x\, y]\end{array}}{\begin{array}{c}\lambda w\lambda t\, \exists xy\, [{}^{0}Have_{wt}\, {}^{0}John\, x] \wedge [{}^{0}Have_{wt}\, {}^{0}John\, y] \wedge\\ [{}^{0}Forged^{*}_{wt}\, x] \wedge [{}^{0}Forged^{*}_{wt}\, y] \wedge [{}^{0}\neq x\, y]\end{array}}}{\lambda w\lambda t\, [{}^{0}Number\_of\, \lambda z\, [[{}^{0}Have_{wt}\, {}^{0}John\, z] \wedge [{}^{0}Forged^{*}_{wt}\, z]] = {}^{0}2]}$$

Types: $Number\_of/(\tau(o\iota))$; $Banknote, Passport, Forged^{*}/\pi$; $Have/(o\iota\iota)_{\tau\omega}$; $Forged/(\pi\pi)$.
There are three conceivable objections to the validity of (*PD*) that we are going to deal with now.

First objection. If Jumbo is a small elephant and if Jumbo is a big mammal, then Jumbo is not a small mammal; hence Jumbo is small and Jumbo is not small. Contradiction!
The contradiction is only apparent, however. To show that there is no contradiction, we apply (*PD*):

$$\frac{\lambda w\lambda t\, [[{}^{0}Small\, {}^{0}Elephant]_{wt}\, {}^{0}Jumbo]}{\lambda w\lambda t\, \exists p\, [[{}^{0}Small\, p]_{wt}\, {}^{0}Jumbo]}$$

$$\frac{\lambda w\lambda t\, [[{}^{0}Big\, {}^{0}Mammal]_{wt}\, {}^{0}Jumbo]}{\lambda w\lambda t\, \exists q\, [[{}^{0}Big\, q]_{wt}\, {}^{0}Jumbo].}$$

Types: $Small, Big/(\pi\pi); Mammal, Elephant/\pi; Jumbo/\iota; p, q \rightarrow \pi$.

To obtain a contradiction, we would need an additional premise; namely, that, necessarily, any individual that is big (i.e., a big something) is not small (the *same* something). Symbolically,

$$\forall w \forall t\, \forall x\, \forall p\, [[[{}^0Big\ p]_{wt}\ x] \supset \neg[[{}^0Small\ p]_{wt}\ x]].$$

Applying this fact to Jumbo, we have:

$$\forall w \forall t\, \forall p\, [[[{}^0Big\ p]_{wt}\ {}^0Jumbo] \supset \neg[[{}^0Small\ p]_{wt}\ {}^0Jumbo]].$$

This construction is equivalent to

$$\forall w \forall t\, \neg\exists p\, [[[{}^0Big\ p]_{wt}\ {}^0Jumbo] \wedge [[{}^0Small\ p]_{wt}\ {}^0Jumbo]].$$

But the only conclusion we can draw from the above premises is that Jumbo is a small something and a big something else:

$$\lambda w \lambda t\, [\exists p\, [[{}^0Small\ p]_{wt}\ {}^0Jumbo] \wedge \exists q\, [[{}^0Big\ q]_{wt}\ {}^0Jumbo]].$$

Hence, no contradiction.

Nobody and nothing is absolutely small or absolutely large, because everybody is made small by something and made large by something else. Similarly, nobody is absolutely good or absolutely bad, everybody has something they do well and something they do poorly. That is, everybody is both good and bad, which here just means being good at something and being bad at something else, without generating paradox.

But nobody can be good at something and bad *at the same thing* simultaneously ($Good, Bad/(\pi\pi)$):

$$\forall w \forall t\, \forall x\, \neg\exists p\, [[[{}^0Good\ p]_{wt}\ x] \wedge [[{}^0Bad\ p]_{wt}\ x]].$$

Second objection. The use of pseudo-detachment, together with an innocuous-sounding premise, makes the following argument valid.

$$\frac{\text{Jumbo is a small elephant} \wedge \text{Mickey is a big mouse}}{\text{Jumbo is small} \wedge \text{Mickey is big.}}$$

$$\frac{\text{If } x \text{ is big and } y \text{ is small, then } x \text{ is bigger than } y}{\text{Mickey is bigger than Jumbo.}}$$

Yet it is not so. We can only infer the necessary truth that if x is a small something and y is a big object *of the same kind*, then y is a bigger object of that kind than x:

$$\forall w \forall t\, \forall x\, \forall y\, \forall p\, [[[[{}^0Small\ p]_{wt}\ x] \wedge [[{}^0Big\ p]_{wt}\ y]] \supset [{}^0Bigger_{wt}\ y\ x]].$$

Additional type: $Bigger/(o\iota\iota)_{\tau\omega}$. This cannot be used to generate a contradiction from these constructions as premises, because $p \neq q$:

$$\exists p\, [[{}^0Small\ p]_{wt}\ a]; \exists q\, [[{}^0Big\ q]_{wt}\ b]$$

Geach, in [4], launches a similar argument to argue against a rule of inference that is in effect identical to (*PD*). He claims that that rule would license an invalid argument. And indeed, the following argument *is* invalid:

> a is a big flea, so a is a flea and a is big; b is a small elephant, so b is an elephant and b is small; so a is a big animal and b is a small animal. (Ibid., p. 33.)

But pseudo-detachment licenses no such argument. Geach's illegitimate move is to steal the property *being an animal* into the conclusion, thereby making a and b commensurate. Yes, both fleas and elephants are animals, but a's being big and b's being small follow from a's being a flea and b's being an elephant, so pseudo-detachment only licenses the following two inferences, $p \neq q$:

$$\exists p\, [[{}^0Big\ p]_{wt}\ a]; \exists q\, [[{}^0Small\ q]_{wt}\ b]$$

And a big p may well be smaller than a small q, depending on the values assigned to p, q.

Third objection. If we do not hesitate to use 'small' not only as a modifier but also as a predicate, then it would seem we could not possibly block the following fallacy:

$$\frac{\begin{array}{c}\text{Jumbo is small}\\ \text{Jumbo is an elephant}\end{array}}{\text{Jumbo is a small elephant.}}$$

$$\frac{\begin{array}{c}\lambda w \lambda t\, \exists p\, [[{}^0Small\ p]_{wt}\ {}^0Jumbo]\\ \lambda w \lambda t\, [{}^0Elephant_{wt}\ {}^0Jumbo]\end{array}}{\lambda w \lambda t\, \exists p\, [[{}^0Small\ {}^0Elephant]_{wt}\ {}^0Jumbo]}$$

But we can block it, since this argument is obviously not valid. The premises do not guarantee that the property p with respect to which Jumbo is small is identical to the property *Elephant*. As was already pointed out, one cannot start out with a *premise* that says that Jumbo is small (is a small something) and conclude that Jumbo is a small B.

4 Conclusion

In this paper, we applied TIL as a logic of intensions to deal with property modifiers and properties in terms of intensional essentialism. Employing the essences of properties, we defined the distinction between non-subsective (that is privative) and subsective modifiers. While the former ones deprive the root property of some but not all of its requisites, the latter enrich the essence of the root property. The main result is the rule of pseudo-detachment together with the proof of its validity for any kind of modifiers.

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