Property modifiers and intensional essentialism

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Adjectives denoting a ***property*** vs. adjectives denoting a function from properties to properties – ***modifiers***

- *Kamp*'s seminal (1975) seeks to draw a line between those adjectives whose meaning is a property and those adjectives whose meaning is a function that maps properties to properties. He (ibid., pp. 147ff) suggests that most adjectives denote a property.
- Yet he admits that it would seem that some adjectives must occur in attributive position and are incapable of occurring in predicative position. They denote a *property-to-property function*
- The same can be said to be true […] of adjectives such as *fake*, *skillful*, or *good*. Where precisely we should draw the boundaries of the class of adjectives to which the second theory [property-to-property function] applies I do not know.
- For example, does *skillful* belong to this class? Surely we must always ask 'skillful what'? before we can answer the question whether a certain thing or person is indeed skillful […]

Taxonomy of modifiers

- ***Modifier M*** is a function of type $(\pi\pi)$; mapping from properties to properties
- **[*M P*]** is a new property that arises from the root property *P* by applying the modifier *M*.
- ***Intersective***. "**A *round* peg is a peg and is round**";
  $[M_i P](a) \therefore M(a) \wedge P(a)$
- ***Subsective***. "**A *skilful* surgeon is a surgeon**";
  $[M_s P](a) \therefore P(a)$
- ***Privative***. "**A *forged* banknote is a *non*-banknote**";
  $[M_p P](a) \therefore \neg P(a)$

Intersective vs. subsective

- The major difference between subsective and intersective modification is that subsectivity bans this sort of argument: $[M_sP](a)$, $Q(a)$ $\therefore$ $[M_sQ](a)$.
- Tilman may be a skillful surgeon, and he may be a painter too, but this does not make him a skillful painter.
- Or, Jumbo may be a small elephant, as well as a mammal, but this does not make Jumbo a small mammal. Jumbo is small *as an elephant* rather than as a mammal.
- ***Scalar adjectives*** **like 'small', 'big' or 'skillful' represent** ***subsective modifiers***.
- On the other hand, to each *intersective modifier* M_i there is a unique 'absolute' property M^* such that if a is an $[M_iP]$ then a is M^* not only as a P but absolutely

Modifiers with respect to a property P

- Modifiers are intersective, subsective or privative *with respect to a property P*. One and the same modifier can be intersective with respect to a property *P* and privative with respect to another property Q.
- For instance, a *wooden table* is wooden and is a table, but a *wooden horse* is not a horse.
- We leave aside the question whether there are modifiers privative with respect to *any* property. Most probably, yes, modifiers like *faked*, *forged*, *false* appear to be privative with respect to any property.

Intensional essentialism

- The essentialism is based on the idea that since *no purely contingent property can be essential of any individual*, *essences are born by intensions* rather than by individuals exemplifying intensions.
- That a property *P* has an *essence* means that a relation-in-extension obtains *a priori* between the property *P* and a set of other properties, the *requisites* of *P*.
- That, for instance, Tilman is a logician is a contingent fact.
- On the other hand, *necessarily*, if Tilman is a logician then he is a man. In other words, being a man is a *requisite* of being a logician.

Requisites and essence

- **Definition** (*requisite relation between ι-properties*)**.** Let *P*, *Q* be constructions of individual properties; $P, Q/*_n \rightarrow (o\iota)_{\tau\omega}$; $x \rightarrow \iota$. Then

$$[^0Req\ Q\ P] = \forall w \forall t\ [\forall x\ [[^0True_{wt}\ \lambda w \lambda t\ [P_{wt}\ x]] \supset [^0True_{wt}\ \lambda w \lambda t\ [Q_{wt}\ x]]]].$$

- Gloss *definiendum* as, "*Q* is a requisite of *P*", and *definiens* as, "Necessarily, at every ⟨*w*, *t*⟩, if it is true that whatever x instantiates *P* at ⟨*w*, *t*⟩ then it is also true that this x instantiates *Q* at ⟨*w*, *t*⟩."

- **Definition** (*essence of a property*). Let $p, q \rightarrow (o\iota)_{\tau\omega}$ be constructions of individual properties, and let $Ess/((o(o\iota)_{\tau\omega})(o\iota)_{\tau\omega})$ be a function assigning to a given property *p* the set of its requisites defined as follows.

$$^0Ess = \lambda p \lambda q\ [^0Req\ q\ p].$$

- Then ***the essence of a property p is the set of its requisites***:

$$[^0Ess\ p] = \lambda q\ [^0Req\ q\ p]$$

Subsectives vs. privatives

A modifier *M* is *subsective* with respect to a property *P* iff

$$[^0\textbf{\textit{Ess}}\ \textbf{\textit{P}}] \subseteq [^0\textbf{\textit{Ess}}\ [\textbf{\textit{M}}\ \textbf{\textit{P}}]].$$

A modifier *M* is *non-trivially subsective* with respect to a property *P* iff

$$[^0\textbf{\textit{Ess}}\ \textbf{\textit{P}}] \subset [^0\textbf{\textit{Ess}}\ [\textbf{\textit{M}}\ \textbf{\textit{P}}]].$$

- a wooden table is a table, but the essence of the property $[^0\textit{Wooden}\ ^0\textit{Table}]$ is enriched by the property of being wooden.
- This property is a requisite of the property of being a wooden table, but it is not a requisite of the property of being a table, because tables can be instead made of stone, iron, etc.

$$[^0\textit{Ess}\ ^0\textit{Table}] \subset [^0\textit{Ess}\ [^0\textit{Wooden}\ ^0\textit{Table}]]$$

Privative modifiers

A modifier *M* is *privative* with respect to a property *P* iff

$$[[^0\textit{Ess}\ P] \cap [^0\textit{Ess}\ [M\ P]] \neq \varnothing \wedge$$

$$^0\exists\lambda p\ [[[^0\textit{Ess}\ P]\ p] \wedge [[^0\textit{Ess}\ [M\ P]]\ \lambda w \lambda t\ [\lambda x\ \neg[p_{wt}\ x]]]]$$

- The modifier *Wooden*/$((o\iota)_{\tau\omega}(o\iota)_{\tau\omega})$ is subsective with respect to the property of being a table, *Table*/$(o\iota)_{\tau\omega}$, but privative with respect to the property of being a horse, *Horse*/$(o\iota)_{\tau\omega}$.
- the property [^{0}*Wooden* ^{0}*Horse*] shares many requisites with the property of being a horse, like the outline of the body, having four legs, etc., and has an additional requisite of being made of wood.

$$[[^0\textit{Ess}\ {}^0\textit{Horse}] \cap [^0\textit{Ess}\ [^0\textit{Wooden}\ {}^0\textit{Horse}]]] \neq \varnothing$$

Privative modifiers

- On the other hand, the modifier *Wooden* deprives the essence of the property of being a horse, *Horse*/$(o\iota)_{\tau\omega}$, of many requisites, for instance, of the property of being an animal, having a bloodstream, a heartbeat, etc.
- Thus, among the requisites of the property [^{0}*Wooden* ^{0}*Horse*] there are properties like *not being a living thing*, *not having a bloodstream*, etc., which are contradictory (not just contrary) to some of the requisites of the property *Horse*.

$$[[^0Ess\ ^0Horse]\ ^0Living\_thing] \wedge$$

$$[[^0Ess\ [^0Wooden\ ^0Horse]]\ \lambda w \lambda t\ [\lambda x \neg [^0Living\_thing_{wt}\ x]]] \wedge$$

$$[[^0Ess\ ^0Horse]\ ^0Blood] \wedge$$

$$[[^0Ess\ [^0Wooden\ ^0Horse]]\ \lambda w \lambda t\ [\lambda x \neg [^0Blood_{wt}\ x]]] \wedge$$

etc.

Privation

- A modifier *M* is *privative* with respect to a property *P* iff the modified property [*M P*] lacks **at least one, *but not all*, of the requisites of the property *P***.

$$[[^0\textit{Ess } P] \cap [^0\textit{Ess } [M\ P]] \neq \varnothing$$

- Furthermore, the modified property [*M P*] has at least one other requisite that contradicts to some of the requisites of *P*.

$$^0\exists\lambda p\ [[[^0\textit{Ess } P]\ p] \wedge [[^0\textit{Ess } [M\ P]]\ \lambda w \lambda t\ [\lambda x\ \neg[p_{wt}\ x]]]]$$

- Corollary

$$[[^0\textit{Ess } P] \cap [^0\textit{Ess } [M\ P]] \subset [^0\textit{Ess } P]$$

$$[[^0\textit{Ess } P] \cap [^0\textit{Ess } [M\ P]] \subset [^0\textit{Ess } [M\ P]]$$

- For instance, a forged banknote has *almost* the same requisites as does a banknote, but it has also another requisite, namely the property of being forged with respect to the property of being a banknote

Privation

- As a result, if M_p is privative with respect to the property *P*, then the modified property $[M_pP]$ and the property *P* are *contrary* rather than contradictory properties:

$$\forall w \forall t\ \forall x\ [[[M_pP]_{wt}\, x] \supset \neg[P_{wt}\, x]] \wedge$$

$$\exists w \exists t\ \exists x\ [\neg[[M_pP]_{wt}\ x] \wedge \neg[P_{wt}\ x]].$$

Adjusted rule for privation

$$\frac{[[M_p\, P]_{wt}\, x]}{[[non\ P]_{wt}\, x]}$$

Pseudo-detachment

- Jumbo is a small elephant → Jumbo is small
- Jumbo is a large animal → Jumbo is large
- ---
- Jumbo is small and large

Pseudo-detachment

- Gamut (the Dutch equivalent of Bourbaki) claims that if Jumbo is a small elephant, then it does not follow that Jumbo is small
- Yet, the conclusion does follow. Jumbo is small *as an elephant*

(1)	*a* is an [*MP*]	assumption
(2)	*a* is an (*M* something)	1, EG
(3)	*M** is the property (*M* something)	definition
(4)	*a* is an *M**	2, 3, SI

Pseudo-detachment

PD, dressed up in full TIL notation, is this:

(PD)

$$\frac{[[MP]_{wt}\ a] \qquad [M^* = \lambda w \lambda t\ \lambda x\ \exists p\ [[Mp]_{wt}\ x]]}{[M^*_{wt}\ a]}$$

- John has a forged banknote and a forged passport
- ---
- John has two forged things

Pseudo-detachment

- *First objection*. If Jumbo is a small elephant and if Jumbo is a big mammal, then Jumbo is not a small mammal; hence Jumbo is small and Jumbo is not small. Contradiction!

$$\frac{\lambda w \lambda t\ [[^0\mathit{Small}\ ^0\mathit{Elephant}]_{wt}\ ^0\mathit{Jumbo}]}{\lambda w \lambda t\ \exists p\ [[^0\mathit{Small}\ p]_{wt}\ ^0\mathit{Jumbo}]}$$

$$\frac{\lambda w \lambda t\ [[^0\mathit{Big}\ ^0\mathit{Mammal}]_{wt}\ ^0\mathit{Jumbo}]}{\lambda w \lambda t\ \exists q\ [[^0\mathit{Big}\ q]_{wt}\ ^0\mathit{Jumbo}]}$$

- $\lambda w \lambda t\ [\exists p\ [[^0\mathit{Small}\ p]_{wt}\ ^0\mathit{Jumbo}] \wedge \exists q\ [[^0\mathit{Big}\ q]_{wt}\ ^0\mathit{Jumbo}]]$
- Hence, no contradiction

Pseudo-detachment

- The conclusion ought to strike us as being trivial. If we grant, as we should, that nobody and nothing is absolutely small or absolutely large, then everybody is made small by something and made large by something else.
- And if we grant, as we should, that nobody is absolutely good or absolutely bad, then everybody has something they do well and something they do poorly. That is, everybody is both good and bad, which here just means being good at something and being bad at something else, without generating paradox
- But nobody can be good at something and bad *at the same thing* simultaneously:
- $\forall w \forall t\ \forall x\ \neg \exists p\ [[[^{0}Good\ p]_{wt}\ x] \wedge [[^{0}Bad\ p]_{wt}\ x]]$

Pseudo-detachment

- *Second objection*. The use of pseudo-detachment, together with an innocuous-sounding premise, makes the following argument valid.

Jumbo is a small elephant $\wedge$ Mickey is a big mouse

Jumbo is small $\wedge$ Mickey is big

If x is big and y is small, then x is bigger than y

Mickey is bigger than Jumbo

Pseudo-detachment

- Yet it is not so. We can only infer the necessary truth that if x is a small something and *y* is a big object *of the same kind*, then *y* is a bigger object of that kind than x:
- $\forall w \forall t\ \forall x\ \forall y\ \forall p\ [[[[^{0}Small\ p]_{wt}\ x] \wedge [[^{0}Big\ p]_{wt}\ y]] \supset [^{0}Bigger_{wt}\ y\ x]]$
- This cannot be used to generate a contradiction from the above premises, because $p \neq q$:
- $\exists p\ [[^{0}Small\ p]_{wt}\ ^{0}Jumbo]$;
- $\exists q\ [[^{0}Big\ q]_{wt}\ ^{0}Mickey]$

Conclusion

- We applied TIL as a logic of intensions to deal with property modifiers and properties in terms of intensional essentialism.
- Employing the essences of properties, we defined the distinction between non-subsective (that is privative) and subsective modifiers.
- Privative modifiers deprive the root property of *some but not all* of its requisites
- Subsective modifiers *enrich the essence of the root property*.
- The main result is the rule of pseudo-detachment together with the proof of its validity for any kind of modifiers.